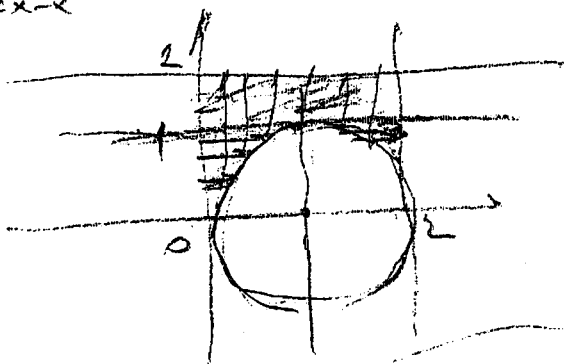


31 (3b) $\int_0^2 dx \int_{\sqrt{2x-x^2}}^2 f(x,y) dy = \int_0^1 dy \int_0^{1-\sqrt{1-y^2}} f(x,y) dx + \int_0^1 dy \int_{1+\sqrt{1-y^2}}^2 f(x,y) dx$ (N)



$$+ \int_1^2 dy \int_0^2 f(x,y) dx$$

$$y = \sqrt{2x-x^2} \quad y \geq 0$$

$$x^2 - 2x + y^2 = 0$$

$$(x-1)^2 + y^2 = 1$$

$$x-1 = \pm \sqrt{1-y^2}$$

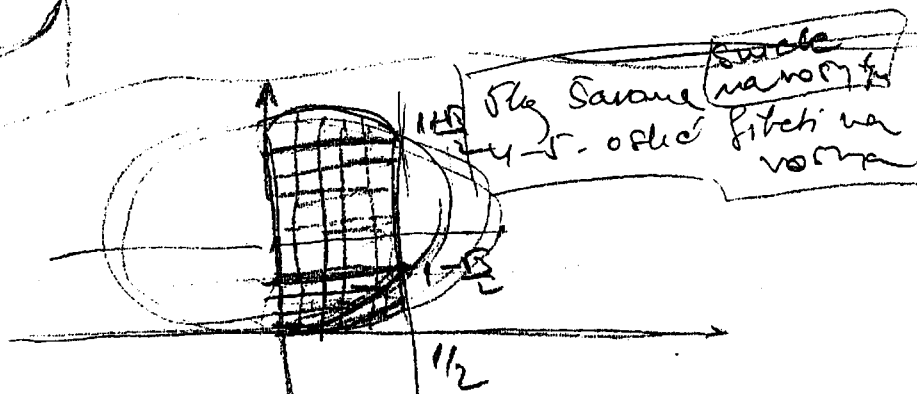
$$x = 1 \pm \sqrt{1-y^2}$$

33 $\int_0^{1/2} dx \int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} f(x,y) dy =$

$$= \int_0^{1-\frac{\sqrt{3}}{2}} dy \int_0^{\sqrt{1-(y+1)^2}} f(x,y) dx$$

$$+ \int_{1+\frac{\sqrt{3}}{2}}^2 dy \int_0^{\sqrt{1-(y-1)^2}} f(x,y) dx +$$

$$+ \int_{1-\frac{\sqrt{3}}{2}}^{1+\frac{\sqrt{3}}{2}} dy \int_0^{1/2} f(x,y) dx$$



$$y = 1 + \sqrt{1-x^2} \quad y-1 = \sqrt{1-x^2} \quad y \geq 1$$

$$x^2 + (y-1)^2 = 1$$

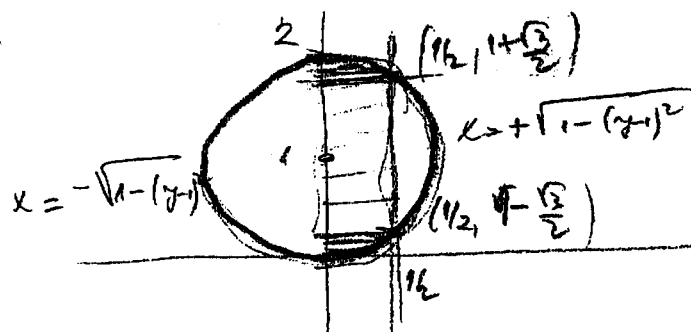
$$x = \pm \sqrt{1-(y-1)^2}$$

$$y-1 = \pm \frac{\sqrt{3}}{2}$$

$$y = 1 \pm \frac{\sqrt{3}}{2}$$

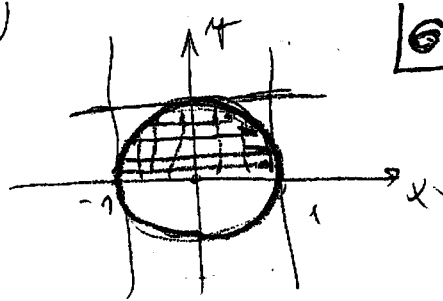
$$x^2 = 1 - (y-1)^2$$

$$x = \pm \sqrt{1-(y-1)^2}$$



25. $\int_{-1}^1 dx \int_0^{\sqrt{1-x^2}} f(x,y) dy$
 $y = \sqrt{1-x^2} / (y \geq 0)$

(1)



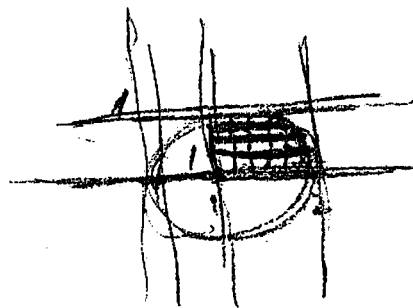
6.3a

$x^2 + y^2 = 1$
 $= \int_0^1 dy \int_{-\sqrt{1-y^2}}^{+\sqrt{1-y^2}} f(x,y) dx$

$x^2 = 1 - y^2$
 $x = \pm \sqrt{1 - y^2}$

27. $\int_{-1}^2 dx \int_0^{\sqrt{2x-x^2}} f(x,y) dy =$

$= \int_0^1 dy \int_{y=\sqrt{2x-x^2}}^{+\sqrt{1-y^2}} f(x,y) dx$
 $y = \sqrt{2x-x^2} / y \geq 0$

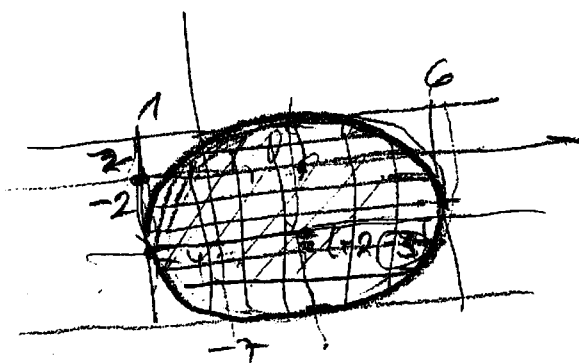


$x^2 + y^2 - 2x = 0$
 $(x-1)^2 + y^2 = 1$
 $(x-1) = \pm \sqrt{1-y^2}$
 $x = 1 \pm \sqrt{1-y^2}$

29. $\int_{-7}^1 dy \int_{2-\sqrt{7-6y-y^2}}^{2+\sqrt{7-6y-y^2}} f(x,y) dx$

$f(x,y) dx = \int_{-2}^6 dx \int_{-3-\sqrt{16-(x-2)^2}}^{-3+\sqrt{16-(x-2)^2}} f(x,y) dy$

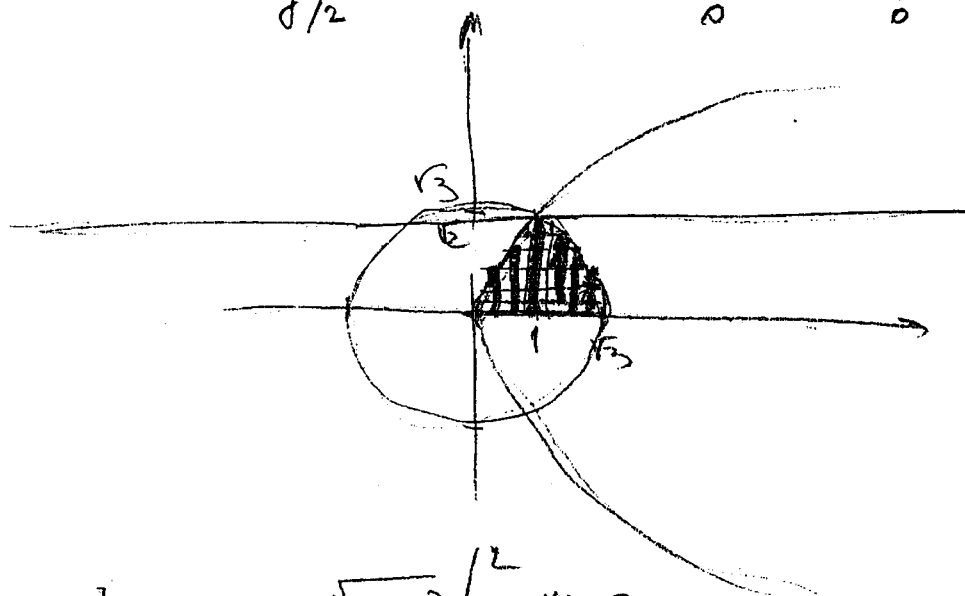
$x = 2 - \sqrt{7-6y-y^2}$
 $\sqrt{7-6y-y^2} = -(x-2) / 2$
 $-(x-2) \geq 0$
 $x-2 \leq 0$
 $x \leq 2$



$(x-2)^2 + y^2 + 6y = 7$

$y = 3 \pm \sqrt{16 - (x-2)^2}$

$$35. \int_0^{\sqrt{2}} dy \int_{\frac{y^2}{2}}^{\sqrt{3-y^2}} f(x,y) dx = \int_0^1 dx \int_0^{\sqrt{2x}} f(x,y) dy + \int_1^{\sqrt{3}} dx \int_0^{\sqrt{3-x^2}} f(x,y) dy$$



$$x = \frac{y^2}{2}$$

$$y^2 = 2x$$

$$y = \pm \sqrt{2x}$$

$$x = \sqrt{3-y^2}/2$$

$$x \geq 0$$

$$x^2 + y^2 = 3$$

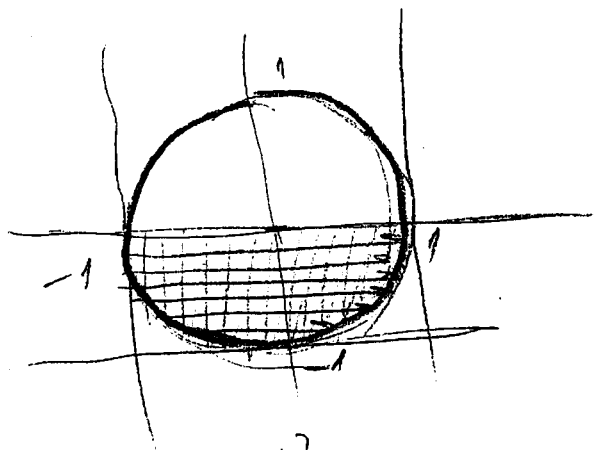
$$y^2 = 3-x^2$$

$$y = \pm \sqrt{3-x^2}$$

$$x^2 + 2x - 3 = 0$$

$$-3 \text{ (1)}$$

$$37. \int_{-1}^1 dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y) dy = \int_{-1}^1 dy \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y) dx$$



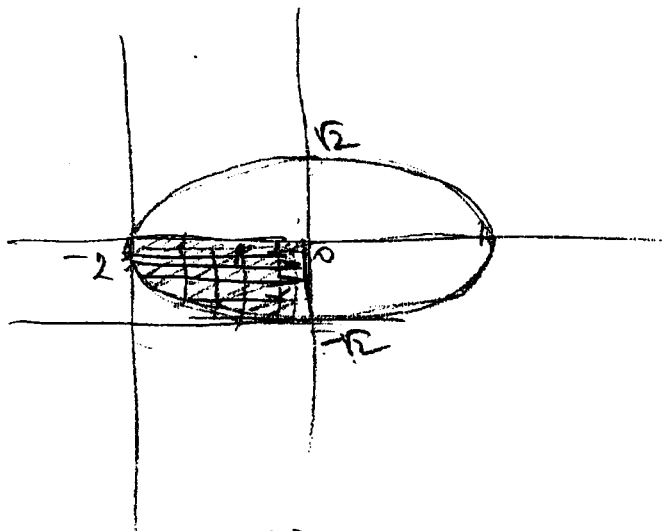
$$-y = +\sqrt{1-x^2}/2, y \leq 0$$

$$x^2 + y^2 = 1$$

$$x^2 = 1-y^2$$

$$x = \pm \sqrt{1-y^2}$$

$$39. \int_{-2}^0 dx \int_{-\sqrt{2-\frac{x^2}{2}}}^0 f(x,y) dy = \int_{-2}^0 dy \int_{-\sqrt{4-2y^2}}^0 f(x,y) dx$$



$$-y = +\sqrt{2-\frac{x^2}{2}} \quad \boxed{y \leq 0}$$

$$\frac{x^2}{2} + y^2 = 2 \quad / : 2 \quad x^2 = 4 - 2y^2$$

$$x = \pm \sqrt{4-2y^2}$$

$$\frac{x^2}{4} - \frac{y^2}{2} = 1$$

$$a^2 = 4 \quad b^2 = 2$$

$$a = 2 \quad b = \sqrt{2}$$

$$41. \int_{-1}^1 dx \int_{-\sqrt{1-x^2}}^{1-x^2} f(x,y) dy = \int_{-1}^0 dy \int_{-\sqrt{1-y^2}}^{+\sqrt{1-y^2}} f(x,y) dx + \int_0^1 dy \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y) dx$$

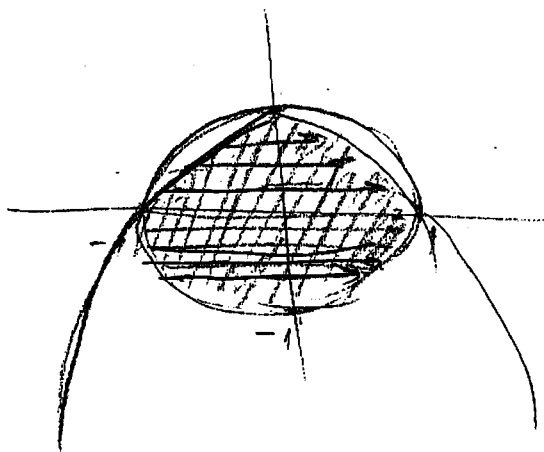
$$-y = +\sqrt{1-x^2} \quad y \leq 0$$

$$x^2 + y^2 = 1$$

$$y = 1-x^2 \quad x^2 = 1-y$$

$$x^2 = 1-y^2 \quad x = \pm \sqrt{1-y^2}$$

$$x = \pm \sqrt{1-y^2}$$



$$45. \int_{-\sqrt{3}}^{\sqrt{3}} dy \int_{1+\sqrt{1+y^2}}^3 f(x,y) dx = \int_2^5 dx \int_{-\sqrt{(x-1)^2-1}}^{\sqrt{(x-1)^2-1}} f(x,y) dy$$

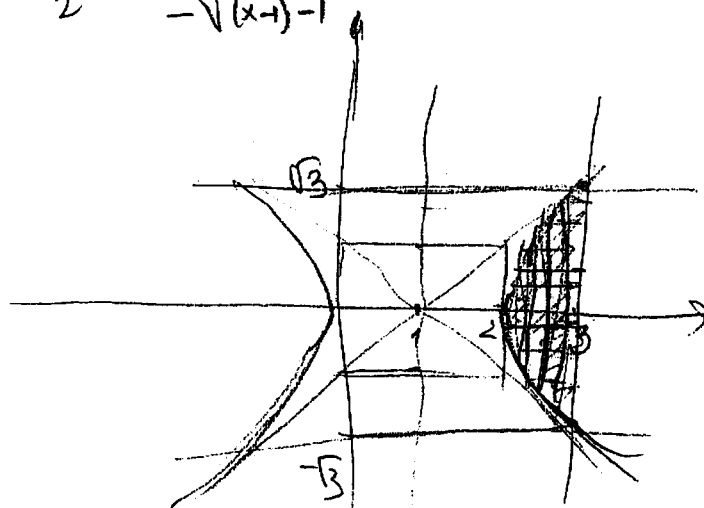
$$x = 1 + \sqrt{1+y^2}$$

$$x-1 = \sqrt{1+y^2} \quad x \geq 1$$

$$(x-1)^2 - y^2 = 1$$

$$y^2 = (x-1)^2 - 1$$

$$y = \pm \sqrt{(x-1)^2 - 1}$$

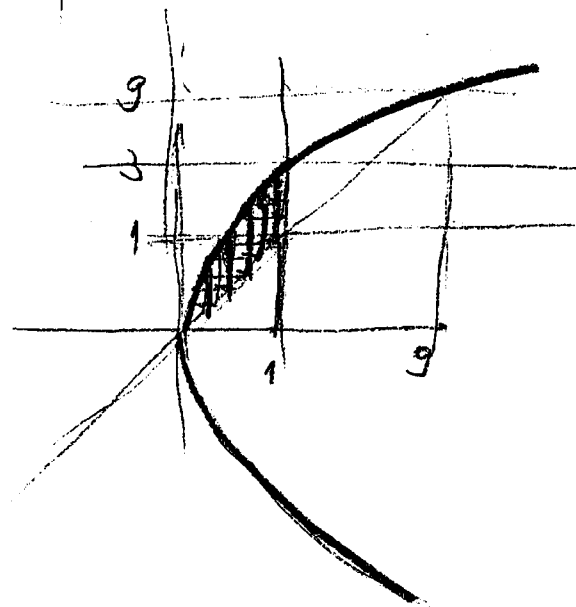
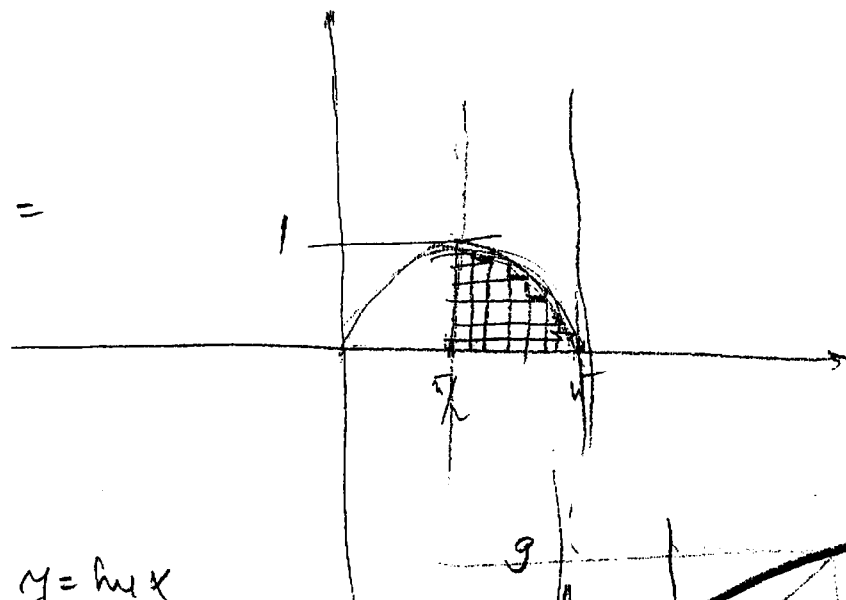


$$49. \int_{\pi/2}^{\pi} dx \int_0^{\tan x} f(x,y) dy =$$

$$= \int_0^1 dy \int_{\arctan y}^{\pi} f(x,y) dx$$

$$y = \tan x$$

$$x = \arctan y$$



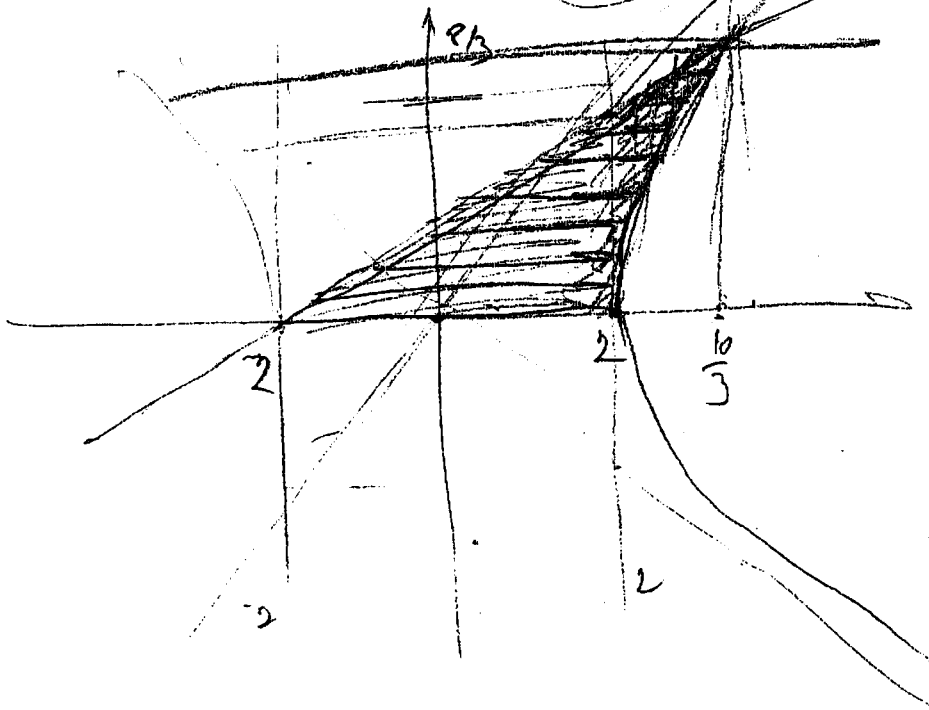
$$53. \int_0^1 dy \int_{y^2/9}^y f(x,y) dx + \int_1^3 dy \int_{y^2/9}^1 f(x,y) dx$$

$$y^2 = 9x \quad = \int_0^1 dx \int_{3\sqrt{x}}^y f(x,y) dy$$

$$y^2 = 9x$$

$$y = 3\sqrt{x}$$

55. $\int_{-2}^2 dx \int_0^{\frac{x+2}{2}} f(x,y) dy + \int_2^{10/3} dx \int_{\sqrt{x^2-4}}^{\frac{x+2}{2}} f(x,y) dy = \int_0^{8/3} dy \int_{2y-2}^{\sqrt{4+y^2}} f(x,y) dx$



$$y = \frac{x+2}{2}$$

$$y = \sqrt{x^2 - 4} \quad y \geq 0$$

$$x^2 - y^2 = 4 \quad / : 4$$

$$\frac{x^2}{4} - \frac{y^2}{4} = 1$$

$$a^2 = 4 \quad b^2 = 4$$

$$a = 2 \quad b = 2$$

$$x^2 - y^2 = 4$$

$$y = \frac{x+2}{2}$$

$$y = \frac{10/3 + 2}{2} = \frac{8}{3}$$

$$x^2 - \frac{x^2 + 4x + 4}{4} = 4$$

$$3x^2 - 4x - 4 - 16 = 0$$

$$3x^2 - 4x - 20 = 0$$

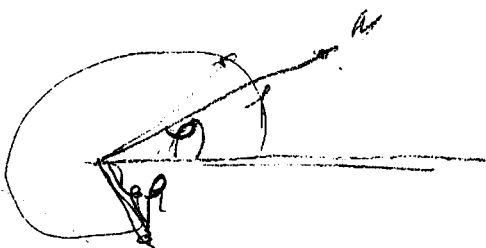
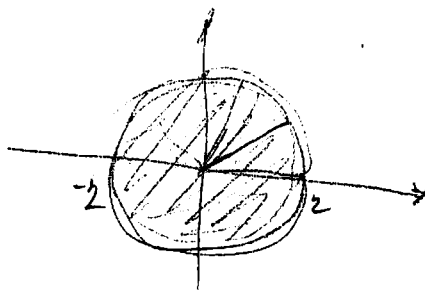
$$x_{1/2} = \frac{4 \pm \sqrt{16 + 240}}{6} < \frac{10}{3}$$

$$-2$$

6.4.A) $x = \rho \cos \varphi, y = \rho \sin \varphi$

6.4.A)

① $D = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 4\}$



$x = f(u,v)$
 $y = g(u,v)$
 $J = \begin{vmatrix} x'_u & x'_v \\ y'_u & y'_v \end{vmatrix}$

$J = \begin{vmatrix} \cos \varphi & -\rho \sin \varphi \\ \sin \varphi & \rho \cos \varphi \end{vmatrix} = \rho \cos^2 \varphi + \rho \sin^2 \varphi = \rho$

$dx dy = \rho d\rho d\varphi$

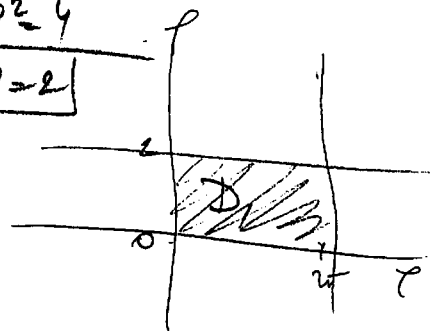
$\iint_D f(x,y) dx dy = \iint_{D_1} f(\rho, \varphi) |J| d\rho d\varphi$

$0 \leq \varphi \leq 2\pi$

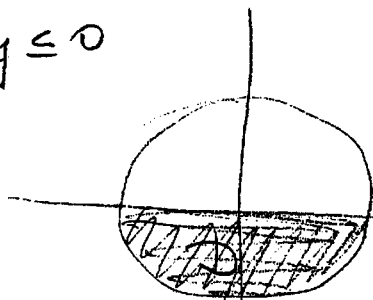
$0 \leq \rho \leq 2$

$\rho^2 \cos^2 \varphi + \rho^2 \sin^2 \varphi = 4$

$\rho^2 = 4$
 $\rho = 2$

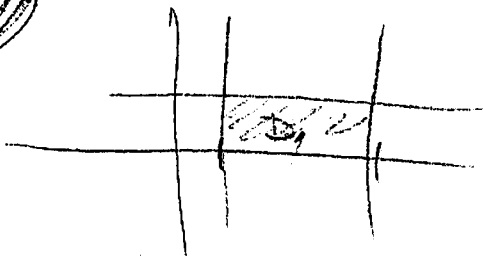


3. $x^2 + y^2 \leq 1, y \geq 0$



$-\pi \leq \varphi \leq \pi$
 $0 \leq \rho \leq 1$

$J = \rho$
 $dx dy = \rho d\rho d\varphi$



$$1. \quad x^2 + y^2 \leq 1 \quad y \geq \frac{1}{2}$$

$$y = 1/2$$

$$x^2 = \frac{3}{4}$$

$$x = \pm \frac{\sqrt{3}}{2}$$

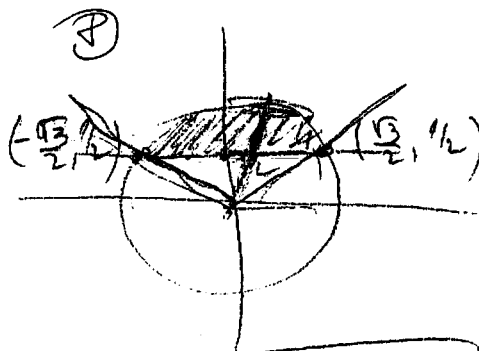
$$\theta_1 = \frac{\sqrt{3}}{2} - \frac{\pi}{3}$$

$$\theta_2 = -\frac{\pi}{3}$$

$$y = 1/2$$

$$r \sin \theta = \frac{1}{2}$$

$$r = \frac{1}{2 \sin \theta}$$

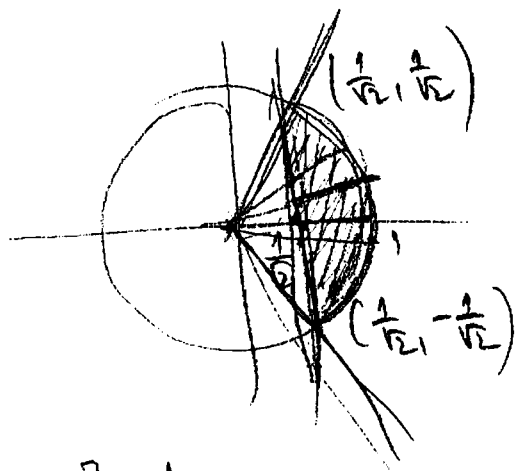


$$\frac{\pi}{3} \leq \theta \leq \frac{2\pi}{3}$$

$$\frac{1}{2 \sin \theta} \leq r \leq 1$$

$$dx dy = r dr d\theta$$

$$2. \quad x^2 + y^2 \leq 1 \quad x \geq \frac{1}{2}$$



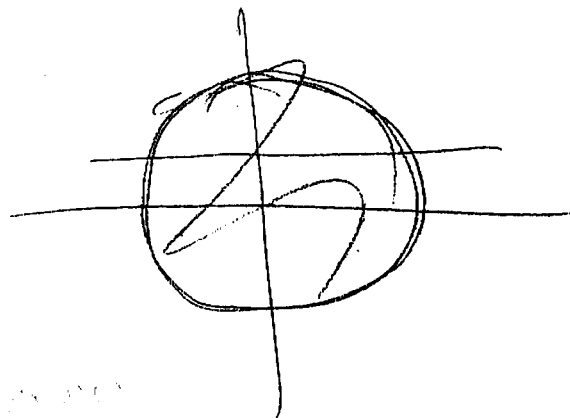
$$x^2 = \frac{1}{4}$$

$$x = \pm \frac{1}{2}$$

$$\theta_1 = \frac{\pi}{4}$$

$$\theta_2 = \frac{3\pi}{4}$$

$$\theta_1 = -\frac{\pi}{4}$$



$$-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$$

$$\frac{1}{\sqrt{2} \cos \theta} \leq r \leq 1$$

$$x = \frac{1}{2}$$

$$r \cos \theta = \frac{1}{2}$$

$$r = \frac{1}{\sqrt{2} \cos \theta}$$

$$dx dy = r dr d\theta$$

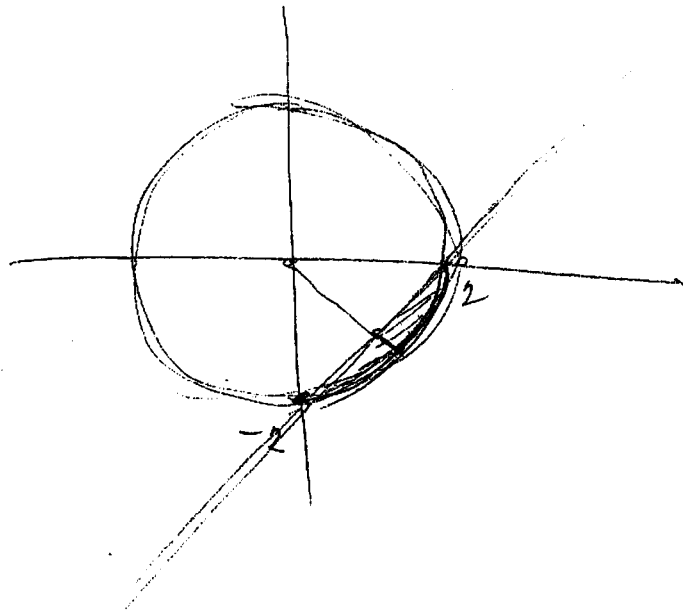
9

9. $x^2 + y^2 \leq 4$ $x + y - 2 \geq 0$

$-\frac{\sqrt{2}}{2} \leq p \leq 0$

min

$\frac{2}{\cos t + \sin t} \leq p \leq 2$



$p \cos t + p \sin t - 2 = 0$

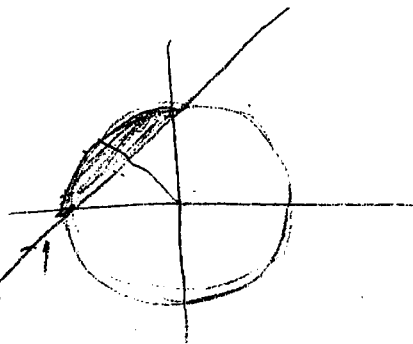
$p = \frac{2}{\cos t + \sin t}$

11. $x^2 + y^2 \leq 1$ $x - y + 1 \leq 0$

$-\frac{\sqrt{2}}{2} \leq p \leq \frac{\sqrt{2}}{2}$ $\frac{1}{\sin t - \cos t} \leq p \leq 1$

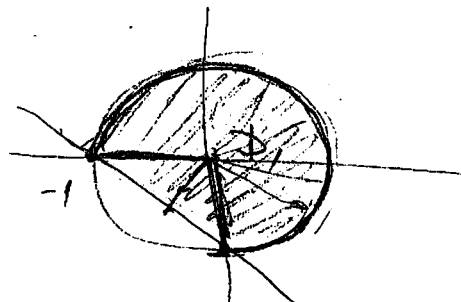
$p \cos t - p \sin t = -1$

$p = \frac{1}{\sin t - \cos t}$



13. $x^2 + y^2 \leq 1$ $x + y + 1 \geq 0$

$-\frac{\sqrt{2}}{2} \leq p \leq \frac{\sqrt{2}}{2}$ $0 \leq p \leq 1$ $0 \leq p \leq \frac{1}{\cos t + \sin t}$



$p \cos t + p \sin t = -1$
 $p = \frac{-1}{\cos t + \sin t}$

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